

## Dispersion Diagram Analysis of a Two-Dimensional Dielectric Hexagonal Periodic Structure

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### Abstract

In this abstract, we present an analysis of the dispersion diagram of a two-dimensional (2-D) dielectric periodic structure with a hexagonal lattice placed in a parallel plate waveguide environment. To highlight the propagation characteristics of the hexagonal structure, we compare it to a periodic structure with a square lattice. The structures are simulated using commercial software, and the results illustrate the attractive wave propagation properties of hexagonal periodic structures, which can offer design flexibility for microwave devices.

## 1 Introduction

Periodic structures are widely used in the design of microwave devices, such as waveguiding systems, filters, antennas, frequency-selective surfaces, metasurfaces, and lenses [1–5]. The use of periodic elements makes it possible to control the wave propagation throughout the structure by modifying the geometry of the periodic inclusions. The analysis of periodic structures always starts with the knowledge of their dispersion diagrams, which relate the propagation constant with frequency. This dispersion diagram is also periodic, and often only a representative repetition (known as the first Brillouin zone) of the propagation constant is reported [6, 7].

Two-dimensional (2-D) periodic structures with rectangular lattices have been extensively studied in microwave and antenna engineering, leading to the design of frequency-selective surfaces, metasurfaces, and more [4, 5]. Although nonrectangular unit cells can provide additional design freedom, little effort has yet been devoted to studying their use in microwave device design. In this work, we focus on the analysis of the dispersion diagram of a hexagonal periodic structure and illustrate the discussion with an electromagnetic simulation of a 2-D dielectric hexagonal periodic structure using commercial software.

## 2 Dispersion Analysis of a Hexagonal Periodic Structure

The dispersion properties of a periodic structure are determined by its geometry and operating frequency. Specifically, the underlying geometrical configuration of the physical lattice, also known as the *direct* lattice, determines the periodicities and symmetries in the dispersion relation. The periodicities of the dispersion relation are typically illustrated using the *reciprocal* lattice, from which the shape of the Brillouin zone can be derived. Further details on the construction and physical meanings of the reciprocal lattice can be found in [6, 7], and elsewhere. In this study, we investigate a dielectric hexagonal periodic structure and present briefly its dispersion properties. To better illustrate its characteristic features, we compare it with the results of a square periodic structure.

Figure 1(a) and Figure 2(a) illustrate the unit cells of the hexagonal and square structures. Both unit cells consist of a dielectric material with a centrally located hole. The dielectric substrate is placed in a parallel plate waveguide (PPW) with top and bottom perfect electric conductor boundary conditions. For a fair comparison, the design parameters were chosen so that the unit cells and holes have the same area. Simulations carried out using *eigenmode solver* in ANSYS HFSS reveal that the hexagonal periodic structure is more isotropic than the square periodic structure, as evidenced by the circular contour lines in the 2-D dispersion diagrams of the fundamental mode shown in Figure 1(b) and Figure 2(b). The isotropic nature of the hexagonal periodic structure is also extended over a wider frequency range compared to that of the square structure, making it a strong candidate for designing microwave devices such as lenses, where high isotropy is highly demanded.

## 3 Conclusion

In this work, we present a preliminary study of the dispersion diagram of a 2-D dielectric hexagonal periodic structure placed in a PPW environment and compare it with a square periodic structure. We used the *eigenmode solver* in ANSYS HFSS to simulate the periodic structures and calculate their 2-D dispersion diagram in the first Brillouin zone. The results provide insight into the dispersion characteristics of hexagonal periodic structures, demonstrating its high level of isotropy compared with the square periodic structure. Further research aims at investigating the potential of hexagonal periodic structures for various applications in the microwave regime.



**Figure 1.** (a) Configuration and design parameters of the 2-D dielectric square unit cell.  $p$  and  $h$  are the side length and height of the unit cell, and  $s$  is the side length of the hexagonal hole located at the center of the unit cell. (b) Isofrequency map (2-D dispersion diagram) of the fundamental mode in the investigated periodic structure. The dimensions of the unit cell used in the simulations are  $p = 2.0$  mm,  $s = 0.875$  mm and  $h = 2.0$  mm. The dielectric material has a relative permittivity of 2.25.



**Figure 2.** (a) Configuration and design parameters of the 2-D dielectric square unit cell.  $p$  and  $h$  are the side length and height of the unit cell, and  $s$  is the side length of the square hole located at the center of the unit cell. (b) Isofrequency map (2-D dispersion diagram) of the fundamental mode in the investigated periodic structure. The dimensions of the unit cell used in the simulations are:  $p = 3.22$  mm,  $s = 1.41$  mm and  $h = 2.0$  mm. The dielectric material has a relative permittivity of 2.25.

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